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Noise Resistant Design of Wavelet Based Multiresolution Control

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Abstract— **In this paper a new method for noise cancellation in a class of wavelet based multiresolution controllers is proposed. Noise cancelation is based on hard thresholding of detail signals in the decomposition stage of the controller. We took advantage of the fact that noise mainly affects high frequency contents of the error signal that is used by the controller, and used hard thresholding to filter their effect in the subsequent control output. The resulting controller outperforms conventional PID and multiresolution PID controllers in closed-loop feedback system design. We tested the performance of the new controller by implementing a n integral plus dead time process used integral of the time-weighted absolute error criteria (ITAE) to tune and compare different controller designs.**

I. Introduction

The wavelet transform, which was first introduced by Mallat [1] more than two decades ago, has been evolved to a useful technique for signal and image processing [2,3], particularly in image compression and denoising [4]. More recently, however, Parvez and Gao [5] used wavelet multiresolution analysis in the design of a new class of controllers which they refered to as multiresolution proportional-integral-derivative (MRPID) controllers. This MRPID decomposes error between the output and set point signals into an approximation and a group of detail signals, which are separated by the frequency information that they contain. After weighting each signal component with an appropriate gain, the control signal as a weighted combination of components. Khan and Rahman [6] successfully implemented MRPID design to control a permanent magnet AC motor system. More recently, the same authors implemented MRPID on a benchmark thermal system [7]. The wavelet property allows decomposition of the signal in both time and frequency domains simultaneously. Short time intervals reveal high frequency components of the signal. On the other hand, long time intervals reveal low frequency components of the signal. The online decomposition of the frequency components can be a powerful technique that can have many practical applications ranging form modeling and control of electromechanical systems, such as electric motor drives, to identification of unknown systems.

In most practical situations, noise is an inevitable player in the control loop that can interfere with the control signal and deteriorate the quality of the output in a closed-loop system, or even cause unstable behavior. Thus, it is essential to take into account the effect of noise in both identification and control of a system. The noise in a closed loop feedback system can stem from different sources, including measurement devices, electrical and mechanical drives and even the plant process. In high precision control, designing a controller that can remove the noise is crucial in guaranteeing the performance. For example in [8] position accuracy of a micro actuator is improved by rejecting narrowband disturbances at bandwidth that were higher that the servo bandwidth of the process. Similarly, Parvez and Gao [5] proposed that decreasing the gain of low scale component of the error signal could improve the noise rejection in the subsequent control signal.

Previously, thresholding has been used for reducition the noise level in images. For instance, Donoho and Johnston [9] used SureShrink thresholding method for denoising images. We took a similar approach in our design wavelet based multiresolution control and applied a threshold to the amplitude of detail components of error signal and subsequently suppressed the effect of noise in the system.

This paper is organized in seven sections. In section two, a brief description of wavelet multiresolution analysis is provided. In section three, the formulation of MRPID as proposed by [5] is described, and the noise rejection scheme as well as modified control design is provided in sections four and five. Next, in section six the performance of the proposed controller is evaluated in an integral plus dead time process model, and finally some conclusions are drown in section seven.

II. Multiresolution Analysis

The multiresolution analysis of a discrete signal is based on decomposing the signal into one approximation and one or more detail signals in different scales. As a case in point, the multiresolution decomposition of a discrete signal f with a samples can be presented in the following form.

$$f \to \left(A_{2^{-J}}^{d} f, \left(D_{2^j} f\right)_{-J<j<0}\right) \quad (1)$$

In (1), J represents the level of signal decomposition, $A_{2^{-J}}^{d} f$ represents the approximation vector, and $D_{2^j} f$ represents the detail vector in different scales ranging from resolution 0 to resolution J. At each level, passing the current approximation vector through a low pass filter derives the next approximation vector. A scaling function defines the low pas filter for each level. Similarly, the detail vector at each level is derived by passing the current approximation vector through a high pass filter, which is defined by the wavelet function at each level. Because the transfer functions for these two filters are complex conjugates, the resulting decompositions are orthogonal. In other words, each vector contains specific information about the original signal that

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cannot be found in other vectors. Single layer multiresolution decomposition of a discrete signal is depicted in fig. 1.

The impulse response for the high pass and the low pass filters at each decomposition stage can be defined as

$$high\ pass\ fileter: g(n) = \langle \psi_{2^{-1}}(u), \phi(u-n) \rangle \quad (2)$$

$$low\ pass\ filter:\ h(n) = \langle \phi_{2^{-1}}(u), \phi(u-n) \rangle \quad (3)$$

In Eq. 2 and 3, ψ(u) and ϕ(u) are wavelet and scaling functions, respectively. Repeating the same process for the resulting approximation vector will result in the next level decomposition of the signal. The decomposition process can continue to infinity, but in practice the application restricts the level of signal decomposition.

Reconstruction of the original signal can be done in two ways, either by passing approximation and details vectors through the pair of conjugate filters namely h' and g' or by creating approximation and detail signals. Combining these signal results in an output signal, namely reconstructed signal, with the same properties as the input signal. The process of creating the decomposition signals for one layer multiresolution analysis is depicted in Fig. 2.

The reconstruction process shown if Fig.2 results in approximation and detail signals that have the same length as the original signal. This is because inserting zeros compensates for the difference in length between input signal and approximation vector or detail vectors at decomposition stage. It is important to note that the wavelet based Multiresolution decomposition of a signal is a linear operation, and the subsequent reconstruction of the original signal is linear. Fig.3 shows different ways to reconstruct the input signal at different level of decomposition.

III. MULTIRESLUTION BASED CONTROLLER DESIGN

Proportional-Integral-Derivative controllers (PID) are widely used for set point tracking, disturbance rejection, and noise rejection in control systems design. Typically, the PID controller has the following form.

$$P(s) = K_C\left(1 + {}^{1}\!/_{\tau_I s} + \tau_D s\right) \times E(s) \quad (4)$$

The integral operator and proportional terms operate in low and medium frequencies of the error signal whereas the derivative term operates in high frequency component of the error signal.

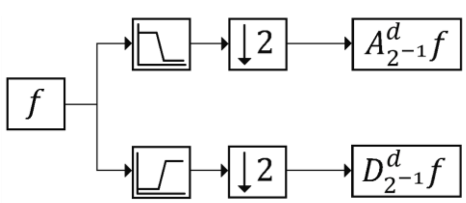


Fig. 1. Single layer decomposition of discrete signal f, after passing the original signal through a high pass and low pass filter, the two resulting vector are down sampled by two to derive the approximation and detail vectors.

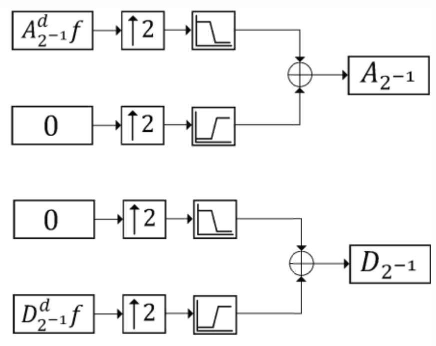


Fig. 2. Creating approximation and detail signals from approximation and detail vectors. After upsampling of vectors, they are passed through a pair of conjugate reconstruction filters to produce the approximation and detail signals.

Frequency division was achieved by decomposing the error signal into different approximation and detail signal. The resulting approximation signal captured low frequency component of the error signal. On the other hand, the detail signals revealed information about the medium to high frequencies. These signals were then weighted based on the desired performance of the system. Combining the weighted approximation and detail signals resulted in the control signal that was applied in the control loop. For the MRPID controller with three level of decomposition, the control signal U is generated by combing one approximation and three detail signals as shown in Eq. 5.

$$U = K_H A_{2^{-3}} + K_{M_2} D_{2^{-3}} + K_{M_1} D_{2^{-2}} + K_L D_{2^{-1}} \quad (5)$$

In Eq. 5, $K_H, K_{M_2}, K_{M_1}, K_L$ are the controller's gains at different scales. For different levels of decompositions, the controller has different number of gains. Two factors must be considered in designing the controller.

A. Type of wavelet and scale functions

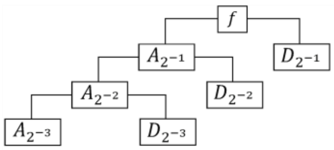
Different types of wavelet and scaling function can be used in decomposition and reconstruction stages. Each family of functions has its own characteristics and specific application. Parvez and Gao in [5] used wavelet function family of Daubechies of order 4 in the design of the controller.

B. Level of decomposition

The level of decomposition is an important factor for designing the controller in order to find the suitable resolution in both time and frequency domains. Eq. 6 can be used for determining the suitable level of the decomposition [5].

$$N \le log_2\left(\frac{2 \times L}{F-1} + 1\right) \quad (6)$$

In Eq. 6, N represents the numbers of controller's gains. In addition, L is the size of the buffer and F is the size of the filter, which is used for decomposition.



$$f = A_{2^{-1}} + D_{2^{-1}}$$
$$= A_{2^{-2}} + D_{2^{-2}} + D_{2^{-1}}$$
$$= A_{2^{-3}} + D_{2^{-3}} + D_{2^{-2}} + D_{2^{-1}}$$

Fig. 3. Different levels of reconstructions from an input signal *f*. In each layer, the approximation signal can be combined with detail signals at current and previous levels to produce the reconstructed signal.

IV. Noise Rejection in Multiresolution PID Control

Parvez and Gao in [5] proposed an approach for noise rejection in the multiresolution PID controller. They reduced the effect of high frequency noise in the output signal by reducing the gain values for the lowest scale. This method reduced the noise dependent fluctuations of output compared to the traditional PID controllers, yet it failed to remove high frequency fluctuations. These high frequency fluctuations can deteriorate the control system performance and lifetime. In this paper, we propose a new method for attenuating these fluctuations. The method is based on thresholding of error signal after the decomposition stage. The resulting controller shows improved performance in cancelling of the noise in different frequency bands. The computations to achieve the noise rejection are minimal, and are described below.

A. Thresholding

Consider a signal $x(n)$ which is contaminated by noise signal $w(n)$. $w(n)$ can be created by many sources, including measurement and environmental noises. Generally, $w(n)$ is considered to be white noise which is statistically independent from $x(n)$. As a result, we can define a noise contaminated signal $f(n)$ as follows.

$$f(n) = x(n) + w(n) \qquad (7)$$

The purpose of denoising is to find an estimate of signal $f(n)$, namely $f'(n)$, such that $f'(n)$ is close to $x(n)$, i.e. the limit of the error between sample of $x(n)$ and $f'(n)$ approches zero. Here we use wavelet multiresolution analysis in order to find $f'(n)$. Consider the signal $f'(n)$ with the wavelet multiresolution decomposition shown in Eq. 8.

$$f' \rightarrow (A^d_{2^{-J}} f', (D_{2^j} f')_{-J<j<0}) \qquad (8)$$

The noise mainly affects the detail vectors in the multiresolution analysis, or higher frequency bands in the original signal. Taking this fact into account, we can manipulate detail vector in order to reduce the effect of noise in the reconstructed signal. Previously, Dohono and Johnstone in [9] proposed a thresholding method for estimating $f'(n)$ close to noise free signal $x(n)$.

Thresholding method suggests two means for denoising the $f'(n)$ signal, hard thresholding and soft thresholding. In both methods it is assumed that the noise affects detail vectors and the changes in the value of the detail vector due to noise is much smaller than that of original signal. Consequently, noise can be attenuated by setting the coefficients that have values smaller than a specified threshold to zero. In this paper we prefer to use hard thresholding method for controller design. Unlike hard thresholding, in soft thresholding all coefficients are subtracted by threshold value, which adds a bias to the all coefficients. As a result, there is a need for tuning the controller's gains so that the performance of the system remains the same in presence of the bias. The hard thresholding method has the following form.

$$T^h_\delta(x) = \begin{cases} x & |x| > \delta \\ 0 & |x| \le \delta \end{cases} \qquad (9)$$

Selection of the threshold value δ is critical in effective denoising of the noise in the controller. Here we use a combination of Stein's Unbiased Estimate of Risk , or SURE method, [9] and Minimax performance which is a fixed threshold [10]. The reason for using the combination of the two method is that in some cases the signal to noise ratio is small and the SURE method maybe unable to cancel noise content of the signal, so there is a need for an alternative method for the selection of threshold value.

V. Design of the Multiresolution PID Control with Enhanced Noise Rejection

Here, we design the control structure for the wavelet Multiresolution PID controller with enhanced noise rejection capability. The controller structure is shown in Fig. 4.

In order to achieve noise rejection, the detail signals pass through a thresholding block after the decomposition stage. The output detail signals are then weighted and combined with the approximation signal in order to form the reconstructed control action signal. In contrast to the noise cancelling method proposed by Parvez and Gao [5], the noise can be attenuated in a wide range of frequencies without the need to change the gains for detail signals. This is due to the fact that in hard thresholding method only the detail coefficients with small values are manipulated. These coefficients are largely caused by noise presented in the system rather than system output. Consequently, using the thresholding has little influence on controller's performance. On the other hand, in the controller structure proposed in [5], changing the gains for reducing the noise effect on detail signals may lead to the poor performance of the controller. In addition, by using the thresholding method high frequency spikes can be obliterated more efficiently. If left unchanged, these spikes in control signal could potentially shorten the efficient lifespan of actuator systems and affect the stability of the closed-loop system. In the next section, we incorporate our proposed controller on a typical model in order to evaluate its performance.

VI. Simulation Results

In this section, we implement our proposed controller on a typical integral plus dead time system to assess the performance of the controller . The system for simulation was

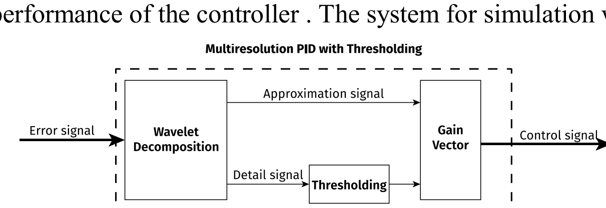


Fig. 4. Controller structure of the wavelet Multiresolution PID controller with enhanced noise rejection.

previously used in [11] to evaluate the performance of their method. The transfer function of the plant system is as follows.

$$P(s) = {0.0506e^{-6s}}/{s} \quad (10)$$

In [11], an enhanced PID controller for this model is proposed. Eq. 11 shows the controller structure.

$$G_c = K_c\left(1 + \frac{1}{\tau_{Is}} + \tau_D s\right)\frac{1 + as}{1 + bs} \quad (11)$$

In the Eq. 11, five different gains need to be tuned for the controller. Shamsuzzoha et al. [11] used the integral of time-weighted absolute error criteria (ITAE) in order to find the best controller's gains. Here, we use the same criteria for tuning both multiresolution PID controller and our proposed controller. Table (1) shows ITAE values for three controllers. Fig. 5 shows the set point response of the controllers.

Next we introduce noise to the system. The noise model is a band-limited white noise with the sampling time of 0.05 seconds. This noise is added to the system as a measurement noise after the system reaches its steady state. Fig. 6 shows the response for three closed-loop systems after the onset of noise in the system.

It is evident from the shape of the outputs that thresholding improves noise rejection for the closed-loop system. Moreover, high frequency fluctuations as well as low frequency fluctuations are removed from the response when the proposed controller is used in the loop.

Table 1. ITAE value for three controllers

Control method	ITAE value
Shamsuzoha and Lee	113.24
MRPID	109.77
Proposed controller	112.91

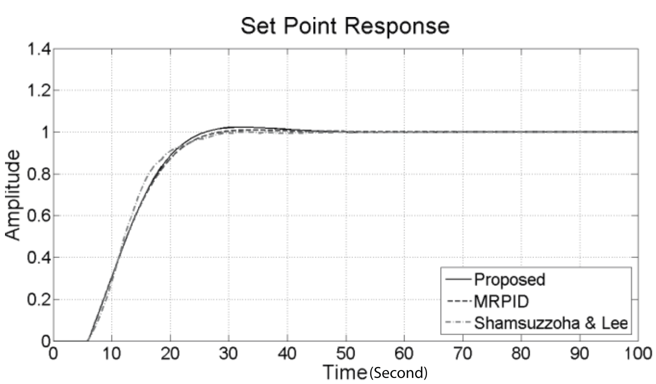


Fig. 5. Set point response for three controllers. They all exhibit fairly the same response to the input signal.

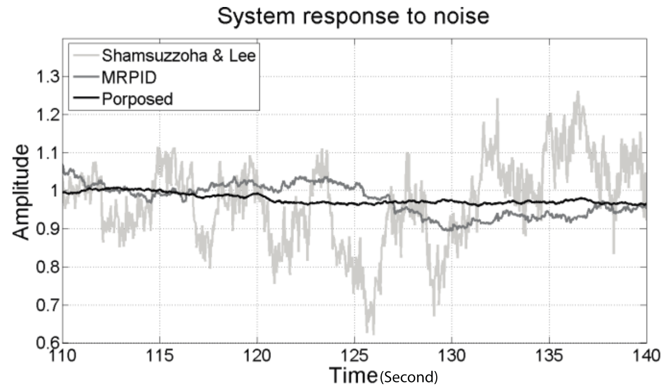


Fig. 6. Response closed-loop system with different controllers in the presence of measurement noise

Based on Fig. 7, in low frequencies, two controllers have fairly similar responses. But as we move to higher frequencies, our proposed method has lower power contents. This means that in higher frequencies the controller act as a low pass filter, thus removes high frequency noises from output.

Control actions signals are also important in a controller design. Two factors should be taken into account, first one is the highest and the lowest amplitude of the output and the second one is the rate the control signals changes from one value to another. We investigate these factors in fig. 8.

Fig. 8 represents an important characteristic of the proposed method and that is the small slew rates of the control signal. In fact, the control signal for MRPID may not be achievable in reality due to high slew rate of control signal. In contrast, the control signal for the proposed method is slow enough to be produced by conventional actuators. The signal has smaller amplitudes compared to the MRPID case. As a result, less amount of energy is needed to remove the noise. This also enhances the performance of the controller and is cost effective.

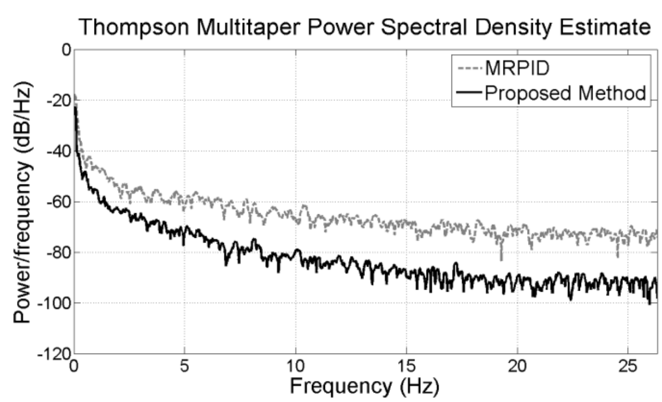


Fig. 7. Power spectral density estimate of Multiresolution PID controller and the proposed method.

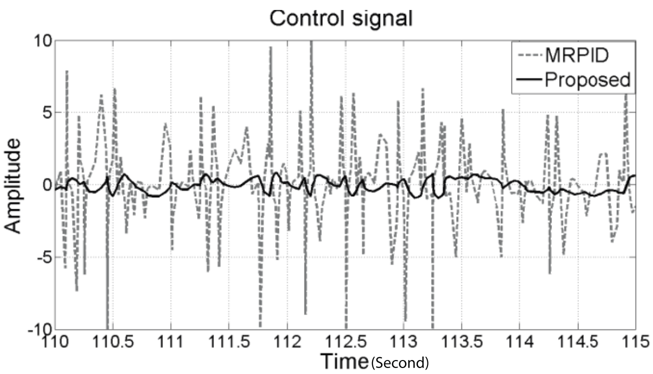


Fig. 8. Control signals for Multiresolution PID controller and the proposed controller.

To clarify the improvement in the performance of the controller, discrete Fourier transform for control signals is computed. Fig. 9 shows the results.

As Fig. 9 shows, in low frequencies two control signals have similar behavior. However, in higher frequencies our proposed method has smaller amplitude in comparison to Multiresolution PID controller. This indicates a smoother control signal, which is essential for the optimal performance of the controller as well as actuators.

VII. Conclusion

In this paper, a new method for enhanced noise canceling ability in Multiresolution wavelet PID controllers is investigated. The performance of the method is evaluated by comparing systems outputs and their corresponding power spectra. In addition, control signal as well as frequency content for each signal is depicted and the superiority of the new method is examined.

Appendix

The controller settings for the simulations in this paper are as follows: Shamsuzzoha and Lee: ($K_c = 1.079, \tau_I = 4.0, \tau_D = 1.5,\ a = 13.197, b = 1.4414$), multiresolution PID ($K_H = 1.5, K_{M_2} = 20, K_{M_1} = 20, K_L = 0$), Proposed: ($K_H = 1.5, K_{M_2} = 20, K_{M_1} = 20, K_L = 0$).

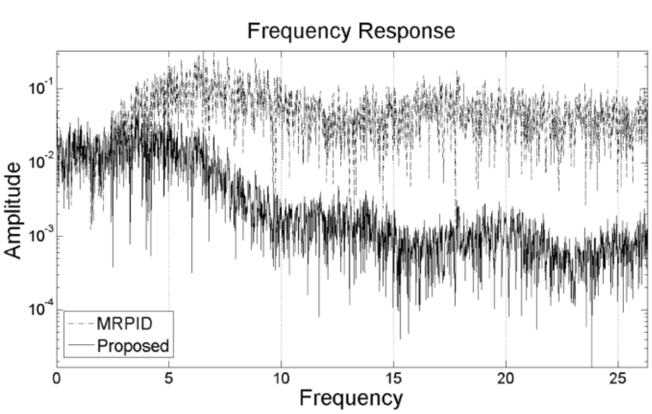


Fig. 9. Frequency contents of the control signal for Multiresolution PID and proposed method

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